Instability of Fluidized Beds

RICHARD C. BAILIE, LIANG-TSENG FAN, and JOE J. STEWART[1]
Kansas State University, Manhattan, Kansas

A related article, "Uniformity and Stability of Fluidized Beds," by these authors appears in *Industrial and Engineering Chemistry*, July 1961, page 567.

A TWO-PHASE system known as the dense phase and the bubble phase (*5*) exists in the gas-solids fluidized bed. The dense phase consists of the solid particles and a portion of the gas held in the interstices of the solids; the bubble phase represents mostly gaseous matter. Formation of the two phases causes gas to bypass the dense phase in the form of bubbles and thus induces the instability of the fluidized bed. This is very undesirable in many processes because it defeats a primary purpose of fluidization—to increase solids-gas contact—and lowers the efficiency of the process.

The purpose of this article is to describe in detail a statistical approach in defining a so-called index of instability of a fluidized bed over the entire range of the bed heights, and the effect of gas velocity, particle size, and bed height upon this index. Similar and related investigations have been conducted by several researchers (*1, 2, 7, 9, 10*).

EXPERIMENTAL METHOD

Statistical Treatment of Data. Density fluctuations in the two-phase, gas-solids, fluidized bed were determined using the radiation attenuation method described by Petrick and Swanson (*8*) and Groshe (*3*). A radioactive γ-ray source provided a beam of γ-radiation which was directed through the center of the fluidizing column. Density fluctuations were determined by detecting, measuring, and recording the portion of γ-radiation which was not attenuated. The count ratemeter for detecting the radiation was equipped with a 30-second and 0.3-second time constant.

Spherical glass beads were used for the fluidized bed. The beads were of two different sizes, 40 to 45 Tyler standard mesh (designated as No. 40 mesh) and 80 to 100 Tyler standard mesh (designated as No. 80 mesh).

The 30-second time constant gave a constant meter reading. The 0.3-second time constant calibrations showed considerable fluctuation and were averaged over 40 readings.

After the density-recorder reading calibration was made, the 0.3-second calibration data were used to calculate the variance (standard deviation squared) in apparent density for packed beds of various average densities. These fluctuations in the apparent density were due to the statistical nature of radioactive decay. A linear regression technique was used to fit the best straight line through the calibration points. The relation used was

$$\sigma_p^2 = A + B\bar{\rho} \tag{1}$$

where A was the variance intercept and B was the slope of the line. The constants A and B from linear regression are tabulated below:

Calibration	A	B
Runs 1–8	0.0004288	0.0006108
Runs 9–38	0.0003812	0.0004915

[1] Present address, North Carolina State College, Raleigh, N. C.

One of the most practical uses of statistics is in the study of variation—i.e., change from a packed to a fluidized bed. But before tests of variance can be used on a sampled population, the sample must prove to have the same distribution upon which the test of variance is based. The simplest and best known distribution is the normal distribution. All of the statistics used in this study of bed uniformity and stability tend to be normally distributed for large samples. From this standpoint, the first step in data analysis was to show that the samples are normally distributed.

Fifty data points were taken from each of six bed conditions. Two of these were on the packed bed, while the remaining four were from beds fluidized with low and high gas velocities. The static bed data were expected to be distributed normally because radioactive decay follows the normal distribution. However, whether the fluidized bed data would be normally distributed was questionable. The results of one of these tests for normality are given in Table I. A 95% confidence level was used and all tests indicated that the sampled populations were normally distributed.

The proof of normality for the sampled population makes it possible to continue the statistical approach. Two useful statistics are the mean and the variance from the mean. The mean is defined as

$$\bar{x} = s_1/N \tag{2}$$

and the variance is the standard deviation squared

$$\sigma^2 = S_2/(N-1) \tag{3}$$

The standard deviation is the estimated standard deviation and is a function of both the variate x_i and the number of observations. For the variate x_i with known standard deviation, σ_{known}

$$\chi^2_{N-1} = \frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{\sigma^2_{known}} = \frac{S_2}{\sigma^2_{known}} = \frac{(N-1)\,\sigma^2_{observed}}{\sigma^2_{known}} \tag{4}$$

where $(N-1)$ is the degrees of freedom, ν, that characterizes the sample. The nearer the values of x_i are to the mean, the smaller will be χ^2. In other words smaller deviations yield smaller χ^2 values. A table is available for this χ^2 distribution with various degrees of freedom (*6*).

The χ^2 statistic, as stated before, is a test to determine how well certain data fit a known or given hypothesis. A 95% confidence level ($p = 0.05$) was used to check the data. This may be interpreted in the following way: Unless the observed value of χ^2 is greater than the χ^2 value given by the χ^2 distribution for a given ν, and $p = 0.05$, there is no reason to suspect the hypothesis being tested; the value of χ^2 may be interpreted as a sampling variation. On the other hand, if the observed χ^2 is greater, indicating that the probability of this occurring is less than 0.05, one must suspect and even reject the tested hypothesis.

This concept is important because the statistic can provide a criterion for deciding whether or not there is a

Table I. Normality Test for Packed Bed

N	x_i	x_i^2	x_i^3	x_i^4
1	8.0	64.00	512.000	4096.0000
2	8.0	64.00	512.000	4096.0000
3	9.0	31.00	729.000	6561.0000
4	2.5	6.25	15.625	39.0625
5	8.6	73.96	636.056	5470.0816
6	8.6	73.96	636.056	5470.0816
7	6.5	42.25	274.625	1785.0625
8	6.0	36.00	216.000	1296.0000
9	5.2	27.04	140.608	731.1616
10	6.0	36.00	216.000	1296.0000
11	10.0	100.00	1000.000	10000.0000
12	8.3	68.84	571.787	4745.8321
13	4.0	16.00	64.000	256.0000
14	8.0	64.00	512.000	4096.0000
15	6.2	38.44	238.328	1477.6336
16	12.1	146.41	1771.561	21435.8881
17	10.0	100.00	1000.000	10000.0000
18	4.0	16.00	64.000	256.0000
19	9.0	81.00	729.000	6561.0000
20	8.0	64.00	512.000	4096.0000
21	6.0	36.00	216.000	1296.0000
22	7.5	56.25	421.875	3164.0625
23	7.8	60.84	474.522	3701.5056
24	6.0	36.00	216.000	1296.0000
25	5.0	25.00	125.000	625.0000
26	4.2	17.64	74.088	311.1696
27	10.1	102.01	1030.301	10406.0601
28	8.0	64.00	512.000	4096.0000
29	7.3	53.29	389.017	2839.8241
30	1.2	1.44	1.728	2.9736
31	6.0	36.00	216.000	1296.0000
32	10.1	102.01	1030.301	10406.0401
33	4.0	16.00	64.000	256.0000
34	5.0	25.00	125.000	625.0000
35	9.0	81.00	729.000	6561.0000
36	6.0	36.00	216.000	1296.0000
37	7.0	49.00	343.000	2401.0000
38	3.0	9.00	27.000	81.0000
39	6.0	36.00	216.000	1296.0000
40	11.2	125.44	1404.928	15735.1936
41	4.5	20.25	91.125	410.0625
42	1.3	1.69	2.197	2.8561
43	4.8	23.04	110.592	530.8416
44	9.0	81.00	729.000	6561.0000
45	6.0	36.00	216.000	1296.0000
46	3.2	19.24	32.768	104.8576
47	6.2	38.44	238.328	1477.6336
48	5.2	27.04	140.608	731.1616
49	5.2	27.04	140.608	731.1616
50	2.1	4.41	9.261	19.4481
s	325.9	2436.27	19892.923	173320.7355
		−2124.22	−47638.820	−518648.2100
			27691.280	621019.7000
				−270737.6700
S		312.05	−54.62	4954.56
k	6.52	6.37	1.16	−15.00
g	0.07	−0.37		
σ_g^*	0.3366	0.6619		
y	0.21	−0.56		

significant difference between the packed bed state and the fluidized state. For instance, let the packed bed be hypothesized and the fluidized bed be observed. The value for χ^2 should exceed the value given by the tables, since it is known that the hypothesis is incorrect. In the event that the observed χ^2 should be less than the statistical value, the data could be rejected because they followed an incorrect hypothesis. Thus the χ^2 test is useful for judging the acceptability of data.

The 0.3-second, time-constant data were analyzed with the statistical approach. The trace made on the strip chart was the trace of the normally distributed fluidized bed population. Because the statistical tests were designed for large samples, 40 data points were taken in each run. From each recorder reading, the corresponding density was read from the calibration curves for the 0.3-second, time-constant data. The 40 densities from the recorder trace were then punched on IBM data cards and loaded into the IBM-650 computer for calculation. For a particular set of 40 densities from the fluidized bed, the computer calculations yielded the following important quantities: the average density, $\bar{\rho}$; the sum of the deviation square, S_2; the variance, σ_p^2, for a packed bed with the same average density as the input data; the ratio of S_2 to σ_p^2 which is χ^2; the variance, σ_f^2, of the fluidized bed,—i.e., the variance of the 40 input densities; and the ratio σ_f^2 to σ_p^2 which was defined as the index of instability.

The χ^2 test was used to determine the acceptability of data. The value for χ^2_{39} at the 5% level is 54.56 (*6*). With the exception of two extreme conditions, data that yielded an observed χ^2_{39} less than this value were rejected for the variance analysis, because a value less than 54.56 meant that the data followed the hypothesis that the bed was packed which was impossible, since all data were taken on the fluidized bed. The two exceptions to this rule were for data taken very near the distributor or in the dispersed phase at the top of the fluidized bed. Data from very near the distributor were not rejected because it was expected that this location would yield small deviations owing to the fact that bubbles had little chance to form (*1*), and the uniformity of fluidization might be great enough to make the bed appear packed. Data from the dispersed phase near the top were also expected to have small deviations and therefore approximate a packed condition. Actually, this state is near an empty column which is a packed condition with average near zero.

Index of Instability. The index of instability (IIS) has been defined as

$$\text{IIS} = \frac{\sigma F^2}{\sigma_p^2} \qquad (5)$$

This is obviously a variance ratio, and at first glance appears to be the F ratio. However, it differs slightly in that the denominator, σ_p^2, is not obtained from a random sample, but from Equation 1, which represents the variance-average density calibration. The variance of the fluidized bed is from a random sampling; therefore, in the strictest sense, IIS is not the F ratio. Because of this, the F-test of significance cannot be used. This in no way effects this study since the χ^2 test is used for a significance test. The IIS compares the unstable fluidized bed to the stable packed bed.

Where the bed is packed, the variance is a result of only the random nature of radioactive decay and is dependent only upon the average density of the bed. In a fluidized bed the variance is due to both the radioactive decay and the disturbance caused by fluidization. The ratio of these variances for a particular average density is then the factor by which the variance is increased due to the fluidizing process.

This index must be defined more precisely, so that its meaning will not be misinterpreted. The terms stability and uniformity have been used in the literature in several ways and need explicit definition.

For a packed bed IIS would be 1. For a fluidized bed disturbances in density are caused by bubbles passing through the bed. These disturbances cause the variance to be greater in a fluidized bed than in a packed bed, which produces an IIS of greater than 1. As the size of the disturbances increases, the instability and thus the IIS increases.

The uniformity of a fluidized bed used here is the rate of change of bed instability with respect to height in the bed. A measure of the nonuniformity is the slope of a curve of IIS *vs.* bed height. The greater the slope, the greater the nonuniformity. These definitions show that it is possible for a bed to be uniformly unstable if the IIS does not change with height.

Table II. Reduced Data for Run 1

H, in.	$\bar{\rho}$	IIS	χ^2_{39}
1.00	1.360	0.58	22.80
1.50	1.345	1.02	39.82
2.00	1.332	0.96	37.68
2.50	1.336	1.03	40.25
3.00	1.321	2.23	87.15
3.50	1.333	1.91	74.68
4.00	1.311	2.47	96.40
4.50	1.320	2.39	93.31
5.00	1.295	3.16	123.47
5.50	1.291	3.10	121.14
6.00	1.316	5.52	215.39
6.25	1.293	4.45	173.79
6.50	1.281	3.67	143.19
6.75	1.285	4.80	187.56
7.00	1.206	7.47	291.69
7.25	0.966	7.60	296.44
7.50	0.637	17.26	673.42
7.75	0.342	12.54	489.38
8.00	0.113	6.96	271.47
8.50	0.093	0.46	18.07
9.00	0.030	0.72	28.22

[a] Packed bed height = 6.65 in., air velocity = 30 ft./min., particle size = 40 mesh.

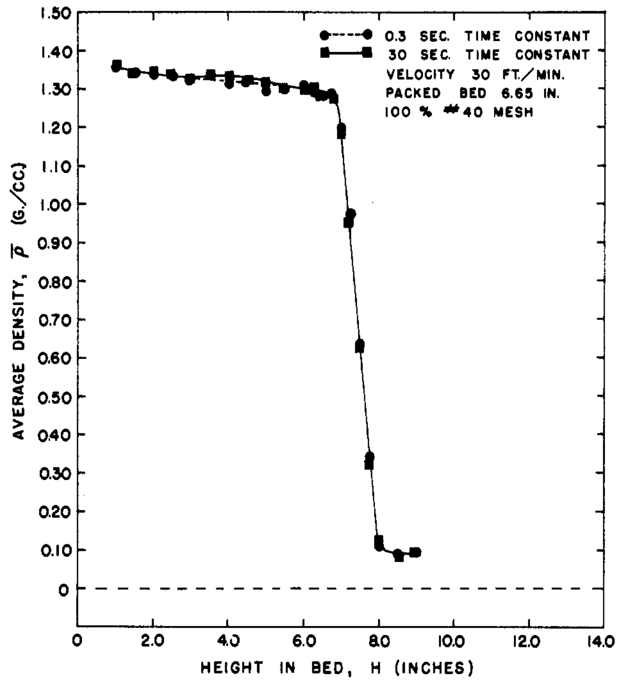

Figure 2. Comparison of long and short time constants

DISCUSSION OF RESULTS

Calibrations and Comparison of Long and Short Time-Constant Data. Many data points were taken so that a good evaluation of various effects could be made. Over 40,000 data points were recorded on strip-chart paper. Because of the extent of the data, only a sample of the reduced data for run number one is presented (Table II).

The spacer technique for density calibration appears to be successful, allowing smooth curves to be drawn through the experimental points. The short and long time-constant calibrations were very nearly the same. This consistency may be regarded as a positive check on the statistical concept of averaging data points.

Figures 1, A, and 1, B, are sample recorder traces for empty and packed beds. The variance was greater for the packed bed. This resulted from the fact that smaller count rates have larger deviations because of a greater attenuation in dense beds. Variance is a function of $1/N - 1$, where N, in this case, is the number of counts or count rate.

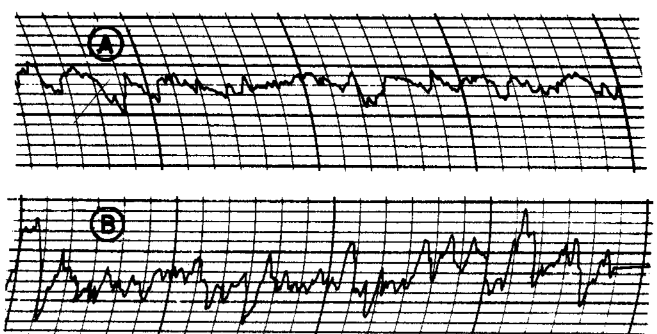

Figure 1. Recorder trace *A* represents the empty bed, *B*, the packed bed

Figure 2 illustrates vertical mean density profiles taken on the 0.3- and 30-second time constants. The results were almost identical, indicating that the calibration was satisfactory, and that either short or long time-constant data could be used to obtain density profiles in a fluidized bed.

The accuracy of 30-second, time-constant data was checked by Lee (*4*), who used the same apparatus as these authors. This was done by measuring the area under the density profile curve, converting it to weight, and comparing it with the known weight of the bed. This material balance method showed that the error was generally less than 5% and in many instances near 1%, indicating that the radiation attenuation method and calibrations were satisfactory.

Operational Variables on Properties of Fluidized Beds. The effects of air velocity and height in bed on the average density are shown in Figure 3. Results are typical; similar results were obtained for all particle mixtures and packed bed heights. The density in section AB of each of the curves decreased with increasing air velocity. This was expected because more air was passing through the bed. With each increase in air velocity the density profiles became more expanded and deviated further from the ideal fluidized bed.

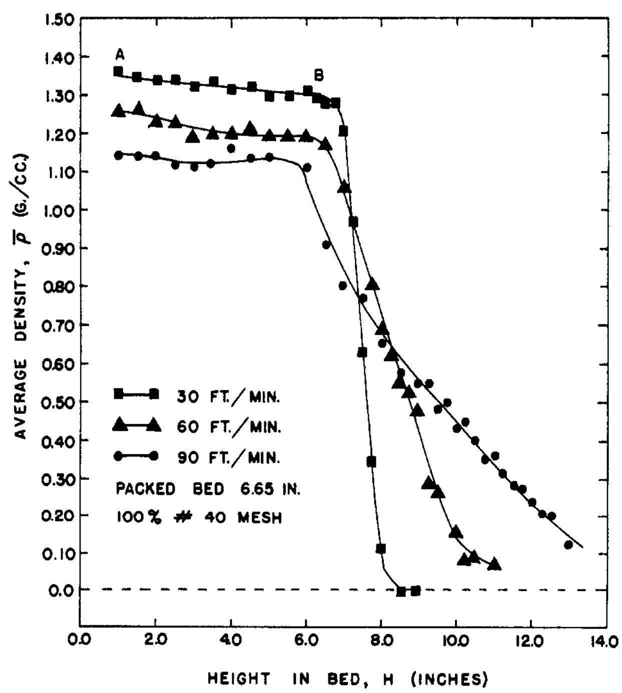

Figure 3. Typical results of air velocity on average density

The effect of air velocity and height in bed on the index of instability (IIS) is presented in Figure 4. The IIS profiles indicate that both stability and uniformity decreased with increasing air velocity. This trend was characteristic of all particle mixtures.

The fact that the lower velocities were more uniform is indicated by the smaller slope of the lower portion of the IIS profiles. The better stability at lower velocities is indicated by the lower magnitude of the index.

The index increased with increasing height above the distributor until a maximum was reached and then dropped abruptly. Assuming that the generally accepted bubble phenomenon is characteristic of the gas-solids system, this trend in the IIS indicates the following mechanisms, which were also suggested by Baumgarten and Pigford (*1*).

As the bubble rises, solids may be carried along with the bubble or forced out of the path of the bubble. It appears as though the rising bubble grows in size as it moves up the column, and it is thus enabled to support the carriage of more solids. As the size increases, the solids mixing becomes more vigorous, and more and more solids must either be forced aside or carried along with the bubble. As the bubble approaches the surface of the bed, the solids above it will be scattered into the empty space above the bed allowing the bubble to break the surface. The solids will then fall back to the bed. The point of maximum disturbance and instability would then be the point where the bubble breaks the surface, because at this point the solids mixing is most vigorous and the bubble is largest. The space just above the bed is disturbed by only the scattered solids that are thrown by the rising bubble. Density fluctuations are, therefore, relatively small above the surface.

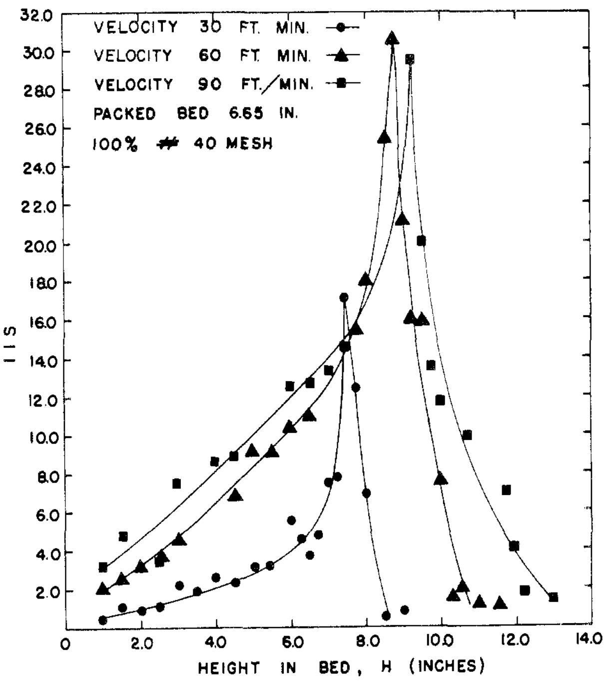

Figure 4. Effect of air velocity on IIS profiles

The shape of the density profiles was found to be related to the IIS profiles. Density profiles became more expanded and less resembled the ideal (perfectly uniform and stable) bed when the air velocity was increased. Also, the stability and unformity decreased with increasing air velocity. The density profiles and IIS profiles could be satisfactorily correlated. This suggests that density fluctuations—bed quality, using a long time constant and studying only the density profile—could be investigated.

In the past there have been some problems in defining the fluidized bed height. If bed expansion is interpreted visually, large discrepancies may result, owing to the constant fluctuations as a result of fluidization. However, the index of instability provides a good criterion for measuring the fluidized bed height.

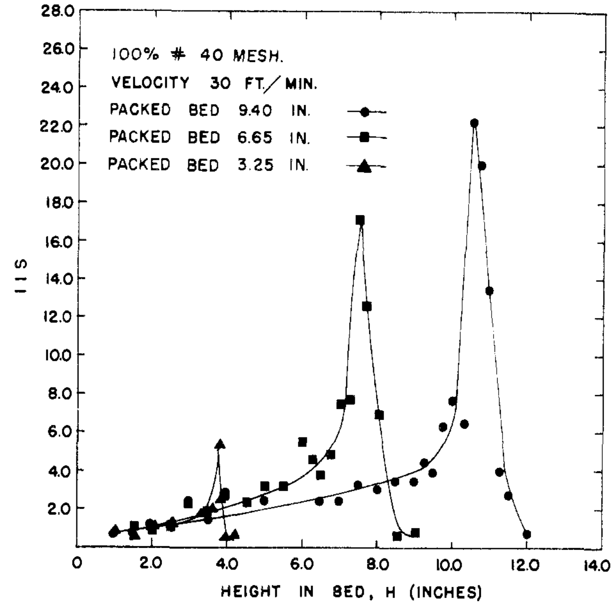

Figure 5. IIS reaches a greater maximum value for a taller bed

Where the bubbles break the surface would be the point of the maximum IIS. This point would also be the fluidized bed height because this is the dense-phase surface.

The inflection point in density-height curve and maximum value of the IIS height curve gave the same value to within 0.5 inch, and either method can be used to obtain the expanded bed height.

Packed bed height does affect the stability and uniformity of the fluidized bed. Figure 5 shows that for a taller bed the instability index of the bed reaches a greater maximum value. However, for small heights the stability of the beds are about the same for all beds. This is in agreement with Dotson and Morse (*2, 7*).

The reason for better uniformity and stability of fluidization in the shallow bed is probably that bubbles have little chance to form and grow in shallow beds.

Figure 6 presents the IIS profiles for fluidized beds of 100% 40 to 45 mesh and 100% 80 to 100 mesh at an air

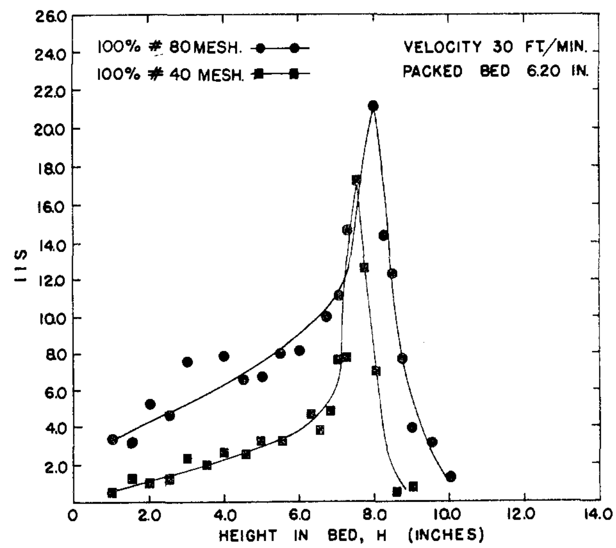

Figure 6. A coarser particle provided a more stable and uniform fluidized bed

velocity of 30 feet per minute. These indicate that the coarser particle size provided both a more stable and a more uniform fluidized bed. As the velocity increased, this trend decreased, and in fact reversed at high velocity. For example, the IIS maxima for the 30 feet per minute velocity were 17.3 and 21.1 for the 40 to 45 mesh and 80 to 100 mesh particles, respectively. At an air velocity of 60 feet per minute the maxima were 30.7 and 30.6, and at 90 feet per minute the maxima were 29.0 and 21.2 for the 40 to 45 mesh and 80 to 100 mesh particles.

The result at the low velocity can probably be explained by the fact that larger particles require a greater fluidizing velocity (*1*). No bubbles can form until the bed is supported by the pressure drop through the interstices. This means that the bubble size will be smaller for coarse particles, since a greater portion of the fluidizing gas is required for minimum fluidization.

Dotson (2) also found that coarse particles produced better uniformity of fluidization at low gas velocity; however, he attributed this trend to the presence of channeling in small particle beds, which he confirmed by visual interpretation.

The reversal of the particle size effect at high velocity may be attributed to factors which oppose the minimum fluidizing effect. Increased permeability in coarse particle beds permits an increased bubble growth and the IIS. This effect becomes more important at high velocities because the amount of gas held in the dense phase is relatively constant and more of the gas flows into the bubble phase (*11*). Reversals similar to this were also obtained by Dotson (2) and Baumgarten and Pigford (*1*).

Figure 7 illustrates the effect of bed composition on the index at low gas velocity. From these profiles it is very difficult to make any precise statement regarding bed

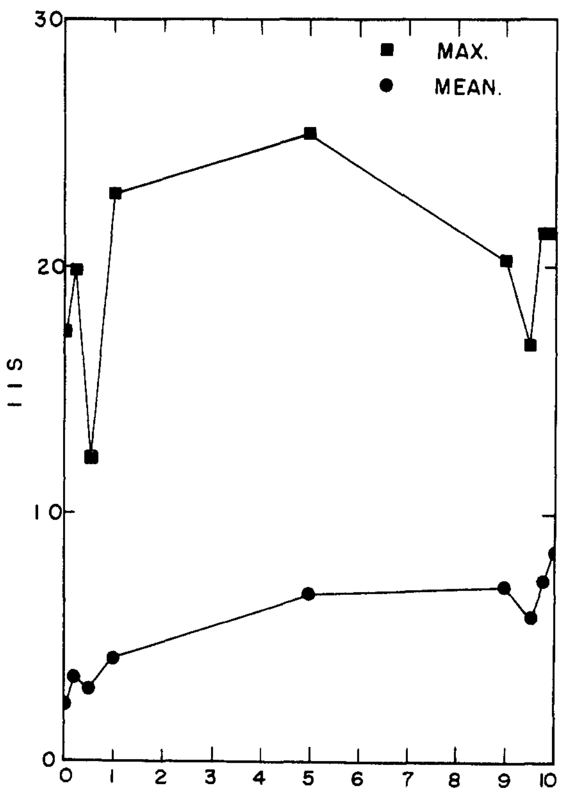

Figure 7. Effect of particle composition on the index at low gas velocity

stability and uniformity. However, the trend is one of relatively good stability and uniformity for particle mixtures with 100, 98, and 95% of 40 to 45 mesh beads. The stability and uniformity decrease for particle mixtures with more than 10% of 80 to 100 mesh beads. For particle mixtures with 50%, or more, of 80 to 100 mesh beads, the stability and uniformity were relatively poor.

To predict better the effects of particle size and bed composition, a wider range of particle sizes should be investigated.

In view of these results, the effect of air velocity and height in bed appears to be much more pronounced than that of particle size and bed composition. This is consistent with the findings of Dotson (*2*).

NOMENCLATURE

A = variance intercept of linear regression
B = slope of line in linear regression
F = variance ratio of the F-test
g = g-statistic
IIS = index of instability
k = k-statistic
N = number of observational data
p = probability
s = sums of variate x_i
S = sums of square of deviations in x_i from mean $\bar{x}$
x_i = variate
$\bar{x}$ = mean (average) value of the variate x_i
y = criterion for significance of test for normality

Greek Letters

γ = gamma radiation
ν = degrees of freedom
ρ = density, g./cc.
$\bar{\rho}$ = average density, g./cc.
σ = standard deviation (estimate)
σ^2 = variance
σ^* = standard error in sampling
χ^2 = chi-square test for goodness of fit

Subscripts

f = fluidized
p = packed
s = spacer
1, 2, 3, 4 = statistics such as s_1, g_2, k_3, etc.

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